KEEPING SECRETS IN THE DIGITAL AGE

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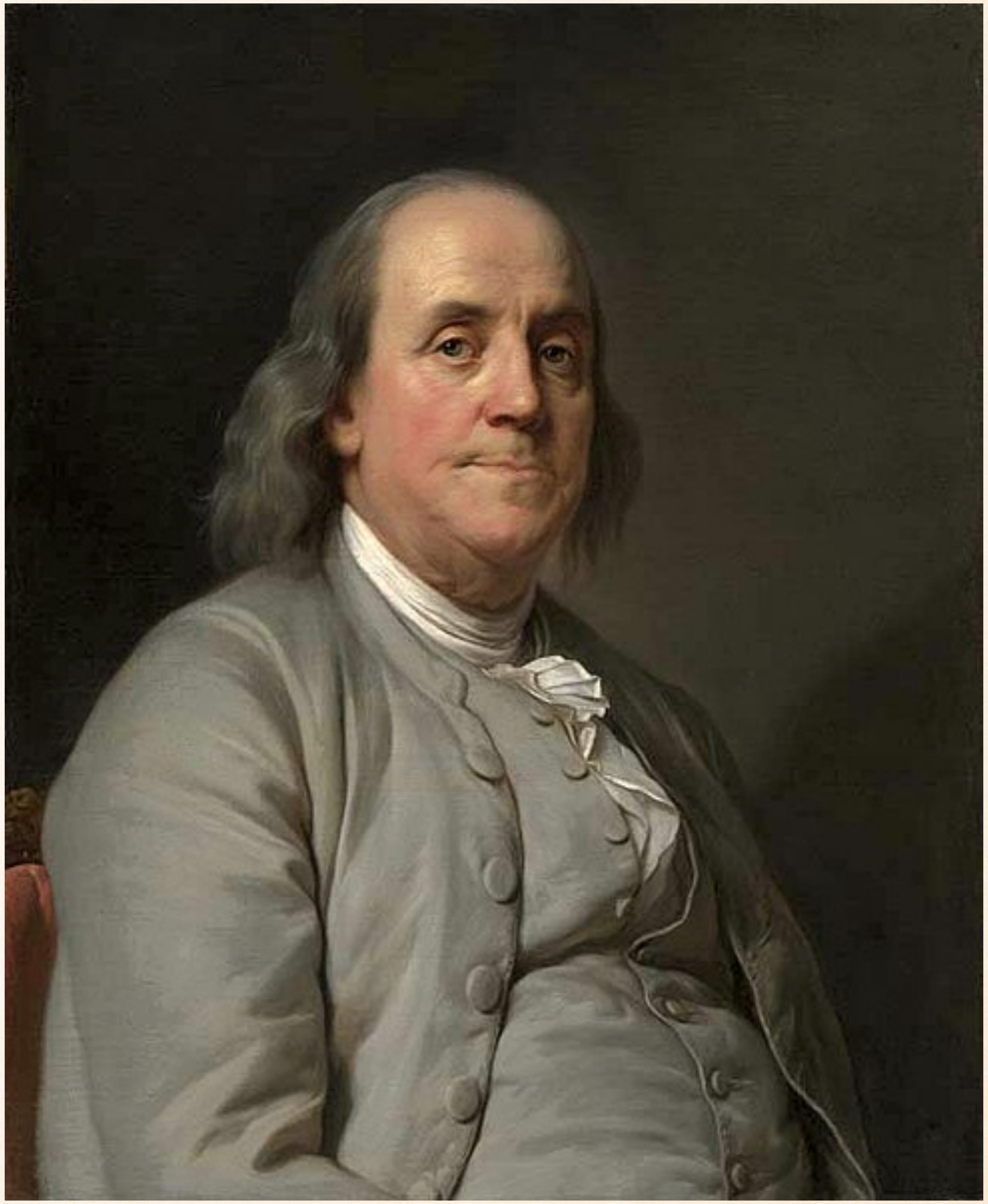
“Three may keep a secret,
if two of them are dead.”

– Benjamin Franklin



- Not really possible to live without secrets.
- Financial details, at the very least!
- Also, state secrets, private communication etc.
- A time-honoured solution: encryption.

ENCRYPTION

- Like putting a document in a safe
- Have no access to document once locked
- Can access only by unlocking using a key

A BRIEF HISTORY OF ENCRYPTION

- Substitution ciphers: Atbash, Mlechhita vikalpa
- Shift/transposition: Caesar cipher, Scytale
- Chinese Nüshu script: secret code for women
- Renaissance: Frequency-based substitution, Vigenère
- World Wars: One time pad, Enigma

EXAMPLE: SUBSTITUTION CIPHER

THIS IS A MESSAGE

HADX DX B SKXXBNK

A	=	B	H	=	A	O	=	O	V	=	L
B	=	V	I	=	D	P	=	Y	W	=	P
C	=	G	J	=	Z	Q	=	F	X	=	U
D	=	Q	K	=	C	R	=	J	Y	=	I
E	=	K	L	=	W	S	=	X	Z	=	R
F	=	M	M	=	S	T	=	H			
G	=	N	N	=	E	U	=	T			

EXAMPLE: SHIFT CIPHER

THIS IS A MESSAGE

+3

WKLV LV D PHVVDJH

MODERN CRYPTOGRAPHY

- Schemes proposed so far are all "symmetric key"
- Both parties must use the same key to encrypt and decrypt
- Needs the key to be securely exchanged before communication! Non-trivial.
- *Public-key cryptography!*

PUBLIC KEY CRYPTOGRAPHY

- Encryption and decryption keys different
- Easier distribution of keys: phone book style
- No need to transport "secret" keys!
- Easy encryption, harder decryption

RSA ALGORITHM

- Find three very large positive integers e, d and $n = p^*q$ for large prime p, q such that for all integers m (with $0 \leq m < n$):

$$((m)^e)^d \equiv m \pmod{n}$$

- Even knowing e and n (or even m), very difficult to find d.

- Need to factorize n into p and q! Considered a hard problem.

- Encrypting key is e, n (public), plaintext message is m, decrypting key is d (private).

OTHER PUBLIC KEY SCHEMES

- Others:
  - Diffie-Hellman-Merkle key exchange
  - ElGamal encryption
- Often slower than symmetric key cryptography.
- Used to exchange symmetric key securely, then symmetric encryption used for communicating!

1970s TO NOW: WHAT'S DIFFERENT?

- Internet is way more ubiquitous
- Text, email, videos — all encrypted! (Hopefully)
- Encrypted data stored on the cloud also

IMPLICATIONS

- Encrypted data is the norm, not the exception
- Schemes so far were “all or nothing”
- Now need a more granular notion of ‘secrecy’!

COMPUTING ON ENCRYPTED DATA

- School: Records are encrypted, principal has key
- Students should know their percentile
- Should not know anyone else's marks!

FUNCTIONAL ENCRYPTION

- Want to be able to compute on encrypted data
- *Functional encryption*
- Can obtain a function of values under encryption

$$\underline{n} = f_{FunE}(\square, \square, \ldots, \square)$$

- Lets student find their percentile, without knowing anyone else's marks

COMPUTING ON ENCRYPTED DATA

- Hospital: Records are (naturally) encrypted, only patient and doctor have the key
- Doctor wants to know the number of her patients who are above 60 years of age
- Like to do the computation on the cloud itself, but records are encrypted
- Nobody else should get access to this info!

HOMOMORPHIC ENCRYPTION

- Functional encryption: anyone can find out result!
- *Fully homomorphic encryption*
- Function on encrypted inputs; result encrypted!

$$\square = f_{FHE}(\square, \square, \ldots, \square)$$

- Only the doctor can decrypt answer to find the actual number of her senior citizen patients.

SO FAR...

- Symmetric-key encryption
- Public-key encryption
- Functional/homomorphic encryption

INFORMATION FLOW

- Information theory (Claude Shannon, 1948)
- Looking at □, should not get *any information* about underlying plaintext
- Given □ and □, should not be able to tell whether same or different plaintext!

INFORMATION FLOW CONTD.

- Symmetric/public-key encryption: Might work
- Functional/homomorphic encryption: Cannot!
  - At least the function computed is known
  - Might use that to distinguish encrypted values
  - Reveals more than plain encryption!

INFORMATION FLOW

- Want to keep “high priority” data from being visible to “low priority” users
- Think of a university database, with many people authorised to do different tasks
- Students might be authorised to see timetable, but not course grades, for example.

INFORMATION FLOW

- Boss fixes a list of salary bonuses for employees
- Wants admin to add them to employee salaries
- Admin who does this should remain anonymous
- Must show proof of being allowed to add bonuses, without revealing identity!
- *Zero-knowledge proofs*

ZERO-KNOWLEDGE PROOFS

- Want to prove a statement to someone *without giving them any further knowledge about it*!
- Want to prove "$x = 1$ or $x = 2$"
- Easy way : Send x, recipient figures out which it is
- NOT a zero-knowledge proof; recipient finds out more than $x = 1$ or $x = 2$!

WHERE'S HOMER?

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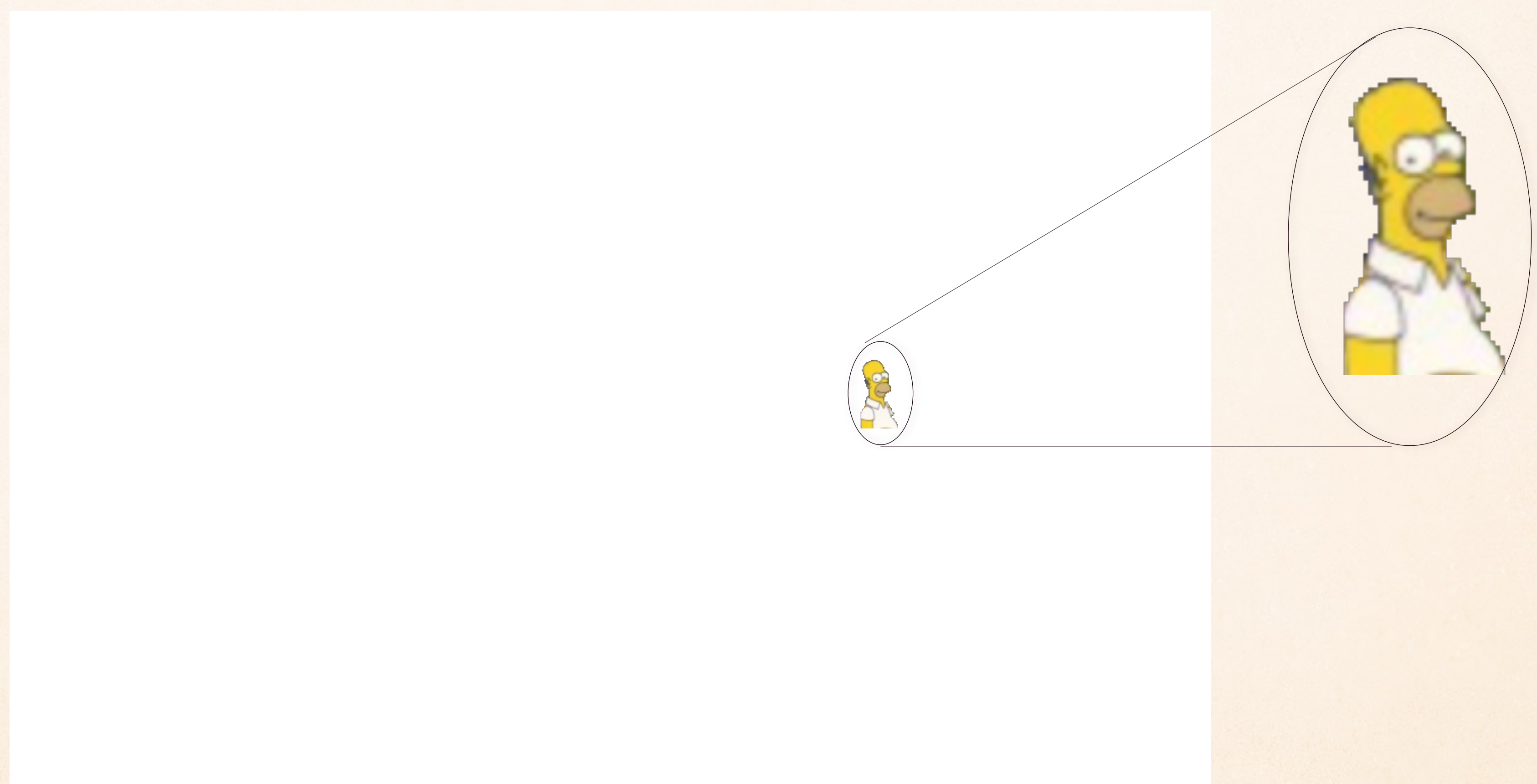
WHERE'S HOMER?



ZERO-KNOWLEDGE PROOFS

- Different ways of implementation
- Depends on the underlying data
- Extend for more granular notions of info-leak
- *Partial-knowledge proofs: quantify how much information allowed to leak*

RECAP

- Symmetric, public-key encryption
- Computing on encrypted data: functional, homomorphic encryption
- Zero/partial knowledge proofs
- Information flow analysis

THANK YOU!

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