
COL876: SPECIAL TOPICS IN FORMAL METHODS

Formal verification of security protocols

Lecture 7, 17 August 2023



QUIZ!

I. Write out the following protocol in the applied-pi calculus.

$$A \rightarrow B : \text{enc}((A, \text{enc}(m, \text{pk}(B))), \text{pk}(B))$$
$$B \rightarrow A : \text{enc}(m, \text{pk}(A))$$

II. Write the equational theory for XOR with identity o . XOR is associative and commutative.

III. When can one get multiple reduction sequences from a single configuration? List out all possible scenarios.

BONUS: Is there a bound on the number of such sequences? If yes, what is it?

RECAP

- Saw how to use ProVerif for automated verification
 - Wrote a simple protocol and properties in ProVerif
 - ProVerif tries to prove a property; if it cannot be proved, tries to produce an attack trace
 - Saw that ProVerif can often produce bizarre error traces, and needs some help to produce a “reasonable” error trace
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PROVERIF: SYNTAX

- Crypto operations specified using equations or *rewrite rules*
 - Declare types, constructor functions, and reduction rules for destructors before starting a protocol description
 - Declare the desired property using the **query** keyword
 - Enriched terms: allow one to include **new**, **if then else** etc in *term syntax*
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ENRICHED TERM SYNTAX

$M, N ::=$	enriched terms
a, b, c, k, m, n, s	names
x, y, z	variables
(M_1, \dots, M_j)	tuple
$h(M_1, \dots, M_j)$	constructor/destructor application
i	natural number ($i \in \mathbb{N}$)
$M + i$	addition ($i \in \mathbb{N}$)
$i + M$	addition ($i \in \mathbb{N}$)
$M - i$	subtraction ($i \in \mathbb{N}$)
$M > N$	greater
$M < N$	smaller
$M \geq N$	greater or equal
$M \leq N$	smaller or equal
$M = N$	term equality
$M \neq N$	term disequality
$M \&\& M$	conjunction
$M \parallel M$	disjunction
$\text{not}(M)$	negation
$\text{new } a : t; M$	name restriction
$\text{if } M \text{ then } N \text{ else } N'$	conditional
$\text{let } T = M \text{ in } N \text{ else } N'$	term evaluation
$\text{event } e(M_1, \dots, M_n); M$	event

PROVERIF: NATURAL NUMBERS

- ProVerif has a type **nat** to represent natural numbers
- One can add and subtract a number i from a term t ($t+i$, $i+t$, $t-i$)
- One can also test for order on terms ($>$, $<$, $>=$, $<=$, $=$, $<>$)
- Constructors cannot have type **nat**, but destructors can

```
type key.
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```
fun enc5(nat, key) : bitstring.
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```
fun dec5(bitstring, key) : nat.
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```
reduc forall x:nat, y:key; dec5(enc5(x+5, y), y) = y.
```

PROVERIF: EQUATIONS

- Suppose I add multiplication over objects of some new type **prd**
- Express that multiplication is commutative? Use **equation**
- Equations need to be convergent (terminating and confluent rewriting) and linear (variables occur at most once in LHS and at most once in RHS)

type prd.

fun mult(prd, prd) : prd.

equation forall x:prd, y:prd; mult(x, y) = mult(y, x).

PROVERIF: STORAGE

- Can model persistent storage using **table**
- Can populate (**insert**) and access (**get**) entries, but not delete
- Tables are not accessible to the attacker

table $d(t_1, \dots, t_n)$.

insert $d(M_1, \dots, M_n)$; P .

get $d(u_1, \dots, u_n)$ **in** P **else** Q .

get $d(u_1, \dots, u_n)$ **suchthat** M **in** P **else** Q .

PROVERIF: PROPERTIES

- Secrecy specified as intruder knowledge: **query attacker(t)**
 - Properties can involve events
 - Correspondence queries specified as implications over events:
query x:t, y:t; event(x, y) ==> event(y, x)
 - What about an event Place-order(t) and an event Cust-pays(t)?
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PROVERIF: PROPERTIES

- Secrecy specified as intruder knowledge: **query attacker(t)**
 - Can check for any reachability property this way
 - Is term M sent on channel c ? **query mess(c, M)**
 - Is (t_1, \dots, t_n) present in table d ? **query table(d(t1, ..., tn))**
 - Does ev occur? **query x1:t1, ..., xn:tn; event (ev(x1, ..., xn))**
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PROVERIF: PROPERTIES

- Correspondence queries specified as implications over events: **query** $x:t; \text{ev1}(x) \implies \text{ev2}(x)$
 - If ev1 has happened, then ev2 has happened (for the same x)
 - What about an event $\text{Place-order}(x)$ and an event $\text{Cust-pays}(x)$?
 - **inj-event** allows us to specify a one-to-one correspondence
 - **query** $x:t; \text{inj-event}(\text{order}(x)) \implies \text{inj-event}(\text{paid}(x))$
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PROVERIF: PROPERTIES

- But what about temporal order?
 - If an order has been placed, the payment was done *before*
 - ProVerif has type **time**; can mark events with timestamp
 - **query** $x:t, i:time, i':time;$
inj-event(order(x))@i ==>
(inj-event(paid(x))@i' && i' < i)
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