COL703 - Proof exercises for system $\mathscr{H}$

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Recall the Hilbert system, with its three axioms and one inference rule.

(H1) $\varphi \supset (\psi \supset \varphi)$

(H2) $(\varphi \supset (\psi \supset \chi)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \chi))$

(H3) $(\neg\varphi \supset \neg\psi) \supset ((\neg\varphi \supset \psi) \supset \varphi)$

$$\frac{\varphi \supset \psi \qquad \varphi}{\psi}\ \text{MP}$$

Prove the following PL expressions in system $\mathscr{H}$. Use DT liberally.

1. $\vdash \varphi \supset \varphi$
2. $\varphi, \varphi \supset \psi \vdash \psi$
3. $\varphi \vdash \psi \supset \varphi$
4. $\varphi \supset \psi, \psi \supset \chi, \varphi \vdash \chi$
5. $\varphi \supset \psi \supset \chi, \varphi \supset \psi, \varphi \vdash \chi$
6. $\neg\varphi \supset \neg\psi, \neg\varphi \supset \psi \vdash \varphi$
7. $\neg\varphi, \varphi \vdash \psi$
8. $\neg\neg\varphi \vdash \varphi$
9. $\varphi \vdash \neg\neg\varphi$
10. $\varphi \supset \neg\psi, \varphi \supset \psi \vdash \neg\varphi$
11. $\varphi \supset \psi, \neg\psi \vdash \neg\varphi$
12. $\neg\varphi \supset \psi, \varphi \supset \psi \vdash \psi$
13. $\varphi, \neg\psi \vdash \neg(\varphi \supset \psi)$