Lecture 2 - Propositional Logic

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COL703 - Logic for Computer Science

Quiz

Recall: Why logic?

- Logic allows us to make sense of our world
- "What constitutes a valid proof?"
- "Is my set of statements internally consistent?"
- Valid inference and internal consistency becomes paramount when we **model complex systems**
- Logic allows us to **verify** that systems work correctly...
- ...without testing each possible execution!
- Important to know when inference is sound!

~~Trust~~ Model, then verify

- A model *abstracts* away extraneous details
- Choice of model heavily tied to the verification context
- Same framework for model and properties we would like to verify
- Sometimes a very simple framework suffices, sometimes not!
- Navigate thin line between expressiveness and tractability of **syntax**
- We start with one of the simplest such: **propositional logic**

Propositional Logic

- Every statement of interest modelled as a **proposition**
- What is a proposition? A statement that can be evaluated for truth or falsehood. Examples:
  - COL703 is a core course for CS5 students
  - New Delhi is the capital of India
  - Blood is gold in colour
- What is not a proposition? Questions, exclamations, doubts...
- Statements whose truth value changes based on context

Compare

- Is there a number such that doubling it and adding two gives ten?
- $2x + 5 = 17$
- See you tomorrow!
- $2 * 4 + 5 = 17$
- $8/0 = 42$
- Hopefully quantum computers will become commonplace soon
- This is not a proposition

Propositional logic: Syntax

- When using a logic, one is bound by the rules of *syntax*
- Only "grammatically-correct" statements are "allowed"
- Start with a (countable) set AP of propositional **atoms**
  - "Smallest" statements of interest
  - Can build up bigger statements with these
- Combine atoms from AP using **operators** to form bigger propositions: AND ($\wedge$), OR ($\vee$), NOT ($\neg$), IMPLIES ($\implies$)
- Grammar for propositional logic (PL) is as follows

$$\varphi, \psi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \implies \psi \qquad \text{where } p \in AP$$