COL352 Problem Sheet 6

Topics: Turing Machines variants and Decidability

Problem 1. (Easy) Prove that a Turing machine with a bi-directional (two-way infinite) tape is equivalent in computational power to a Turing machine with a uni-directional (one-way infinite) tape.

Problem 2. (Medium) A write-once Turing machine is a single-tape Turing machine that can modify each tape cell at most once (including cells containing the input). Show that the write-once Turing machine model is equivalent in computational power to the standard Turing machine model.

Hint: Consider first showing that a write twice TM is equivalent to standard TM

Problem 3. (Medium) An oblivious Turing machine is a Turing machine whose head movement depends only on the length of the input and not on the contents of the tape. In other words, it follows a fixed pattern of left-to-right and right-to-left sweeps independent of the input symbols. Prove that every Turing machine can be simulated by an oblivious Turing machine.

Hint: Consider a k-tape where we place head markers and scan the whole input to find symbol at tape heads

Problem 4. (Medium) A multi-tape Turing machine has multiple tapes, each with its own head. Prove that multi-tape Turing machines are equivalent in computational power to single-tape Turing machines.

Hint: Consider using Γ^k as your tape alphabet when simulating k-tape machine on 1-tape machine

Problem 5. (Medium) A multi-head Turing machine is a single-tape Turing machine with multiple heads operating on the same tape. Show that multi-head Turing machines are equivalent in computational power to standard single-head Turing machines.

Hint: Consider using $\Gamma \cup \dot{\Gamma}$, where $\dot{\Gamma}$ denotes marked alphabet $\dot{a}$, .. when simulating k-head with 1-head

Problem 6. (Medium) A nondeterministic Turing machine (NTM) allows multiple possible transitions from a given configuration. Prove that nondeterministic Turing machines are equivalent in computational power to deterministic Turing machines.

Hint: Consider performing BFS over the tree of NTM states when simulating with DTM

Problem 7. (Hard) A Universal Turing Machine (UTM) is a Turing machine that takes as input the encoding of another Turing machine (M) and a string (w), and simulates (M) on (w). Prove that there exists a Universal Turing Machine.

Problem 8. (Hard) Define a linear bounded automaton (LBA) as a Turing machine whose tape is restricted to the portion initially containing the input. Show that LBAs are strictly less powerful than general Turing machines. *Hint: Create a Problem needs more than linear space.*

Problem 9. (Medium) A Turing machine with a two-dimensional tape has its tape arranged as an infinite grid. Prove that this model is equivalent in computational power to the standard one-dimensional tape Turing machine.

Hint: Consider mapping tape positions from 2D tape to 1D using a bijective map from $f : \mathbb{N}^2 \to \mathbb{N}$

Problem 10. (Medium) Consider a Turing machine where the tape head can move to the right or stay put, (R,S instead of R,L). Show that the set of languages recognized by this type of TM is the set of regular languages.

Hint: Try showing the lang. has finitely many equivalent classes hence by Myhill-Nerode it is a regular lang.

Problem 11. (Medium) For each of the following languages, determine whether it is decidable or undecidable.

1. $EQ_{DFA} = \langle A, B\rangle \mid A, B$ are DFAs and $L(A) = L(B)$
2. $EQ_{NFA} = \{\langle A, B\rangle \mid A, B$ are NFAs and $L(A) = L(B)\}$
3. $EQ_{REX} = \{\langle R_1, R_2\rangle \mid R_1, R_2$ are regular expressions and $L(R_1) = L(R_2)\}$
4. $ALL_{DFA} = \{\langle D\rangle \mid D$ is a DFA and $L(D) = \Sigma^*\}$
5. $ALL_{CFG} = \{\langle G\rangle \mid G$ is a CFG and $L(G) = \Sigma^*\}$
6. $EQ_{CFG} = \{\langle G_1, G_2\rangle \mid G_1, G_2$ are CFGs and $L(G_1) = L(G_2)\}$
7. $AMB_{CFG} = \{\langle G\rangle \mid G$ is an ambiguous CFG$\}$
8. $REG_{CFG} = \{\langle G\rangle \mid G$ is a CFG and $L(G)$ is regular$\}$
9. $HALT_{TM} = \{\langle M, w\rangle \mid M$ is a TM that halts on input $w\}$
10. $EMPTY_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M) = \phi\}$
11. $FIN_{TM} = \{\langle M\rangle \mid M$ is a TM and $L(M)$ is finite$\}$
12. $INF_{CFG} = \{\langle G\rangle \mid G$ is a CFG and $L(G)$ is infinite$\}$

Problem 12. (Medium) For each of the following languages, determine whether it is decidable or undecidable.

1. $T = \{\langle M\rangle \mid$ M is a TM that accepts w^R whenever it accepts w$\}$.
2. $T = \{\langle M\rangle \mid$ M is a DFA that accepts w^R whenever it accepts w$\}$.
3. $T = \{\langle M\rangle \mid$ M is a DFA that accepts some palindrome$\}$.
4. $T = \{\langle G, H\rangle \mid$ G and H are CFGs where L(G) $\cap L(H) \neq \phi\}$.
5. Say that a variable A in CFG G is necessary if it appears in every derivation of some string w $\in$ G. Let $NECESSARY_{CFG} = \{\langle G, A\rangle \mid$ A is a necessary variable in G$\}$.