COL352 Problem Sheet 5

Topics: Turing Machines Properties, Decidability

The number of cells a TM uses other than input is its SPACE Complexity, and number of steps it takes to terminate is its TIME Complexity.

Problem 1. (Easy) Show that a pushdown automaton (PDA) with $n \geq 2$ stacks is equivalent in computational power to a Turing machine.

Problem 2. (Easy) Answer the following questions about Turing machines:

1. Can the description of a Turing machine be written as a string?
2. Is the number of Turing machines finite, countable, or uncountable?
3. Can infinitely many string represent the same TM ?

Problem 3. (Easy) Given binary representations of x and y as input to a Turing machine, describe how the machine would compute the following:

1. $x - 1$
2. $x \times y$
3. x^y

Problem 4. (Easy) Show that all the non-context-free languages discussed in the previous tutorial can be decided by a Turing machine.

Problem 5. (Easy) If TM M, M' computes $f(x), g(x)$ for input x. Construct a TM, which can compute the following:-

1. $f(g(x))$
2. $f^n(x)$ where n is given as input.

Problem 6. (Easy) Prove that the following languages are decidable:-

1. $A_{DFA} = \{\langle B, w\rangle | B \text{ is a DFA that accepts } w\}$

2. $A_{NFA} = \{\langle B, w\rangle | B \text{ is a NFA that accepts } w\}$
3. $A_{REX} = \{\langle R, w\rangle | R \text{ is a regular expression generates } w\}$
4. $E_{DFA} = \{\langle D\rangle | D \text{ is a DFA and } L(D) = \phi\}$
5. $E_{DFA} = \{\langle A, B\rangle | A, B \text{ are DFA and } L(A) = L(B)\}$
6. $A_{CGF} = \{\langle C, w\rangle | C \text{ is a CFG that generates } w\}$
7. $E_{CFG} = \{\langle D\rangle | D \text{ is a CFG and } L(D) = \phi\}$