COL352 Problem Sheet 3

February 13, 2025

Problem 1. Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context free.

Problem 2. Show that the language $A = \{w | w \in \{a, b, c\}^*$ and contains equal number of a's, b's, and c's *is not a CFL*.

Problem 3. Let $L = \{w | wx \in A \text{ for some } x \in B\}$. Show that if A is context-free and B is regular, then L is context-free.

Problem 4. Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many a's as b's.

Problem 5. Let $L = \{x \# y | x, y \in \{0, 1\}^* \text{ and } x \neq y\}$. Show that L is a context-free language.

Problem 6. Give unambiguous CFGs for the following languages.

- 1. $\{w \mid in every prefix of w the number of a's is at least the number of b's\}$
- 2. $\{w \mid \text{the number of } a's \text{ and } b's \text{ in } w \text{ are equal}\}$
- 3. $\{w \mid \text{the number of } a's \text{ is at least the number of } b's\}$

Problem 7. Let $L = \{xy \mid x, y \in \{0, 1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}$. Show that L is a context-free language.

Problem 8. Let $L = \{a^i b^j | i \neq j \text{ and } 2i \neq j\}$. Show that L is a context-free language.

Problem 9. Consider the grammar $G = (\{S, Z, O\}, \{0, 1\}, R, S)$ described by the rules: $R = \{ZZ \longrightarrow \epsilon, OOO \longrightarrow \epsilon, ZO \longrightarrow OOZ, OZ \longrightarrow ZOO, O \longrightarrow 1, Z \longrightarrow 0 \}$

Given the Grammar $G(s) = (\{S, Z, O\}, \{0, 1\}, R \cup \{S \longrightarrow s\}, S)$ where $s \in \{Z, O\}^*$, provide an efficient algorithm to obtain the length of the shortest word in $\mathcal{L}(G(s))$.