Recognizability And Decidability

Recall: A Turing machine is essentially an FSA with infinite tape
The tape head can move left or right
$M = (Q, \Sigma, \Gamma, \delta, s, t, r)$ is a deterministic TM
Start in s with the input on the tape, tape head at its first letter
What changes with each letter of the input?
Current state, tape contents, tape head position
These form a configuration: uqv
$u, v \in \Gamma^*$, $q \in Q$, tape head at first letter of v.

Let $u, v \in \Gamma^*$, and $a, b, c \in \Gamma$. Then,

$uaqbv \xrightarrow[M]{1} uacq'v$ iff $\delta(q, b) = (q', c, R)$, and

$uaqbv \xrightarrow[M]{1} uq'acv$ iff $\delta(q, b) = (q', c, L)$

u, v generally taken to be the two "halves" of the input around head, bookending $\sqcup$ symbols ignored.

Saw an example last time where $0111001 q_2 \xrightarrow[M]{1} 011100t1$

We had $\delta(q_2, \sqcup) = (t, \sqcup, L)$.

So the shape of the configuration change should be

$uaqbv \xrightarrow[M]{1} uq'acv \qquad q = q_2, \quad q' = t$

What are u, v, a, b, c?

$u = 011100 \quad a = 1 \quad b = \sqcup \quad v = \varepsilon$

$uqv \rightsquigarrow (\sqcup)^* uqv (\sqcup)^*$

What is the "language of a Turing machine" $M = (Q, \Sigma, \Gamma, \delta, s, t, r)$?

Consider $P_M = \{ \omega \mid \text{there are } u, v \in \Gamma^* \text{ s.t. } s\omega \xrightarrow[M]{*} utv \}$, and

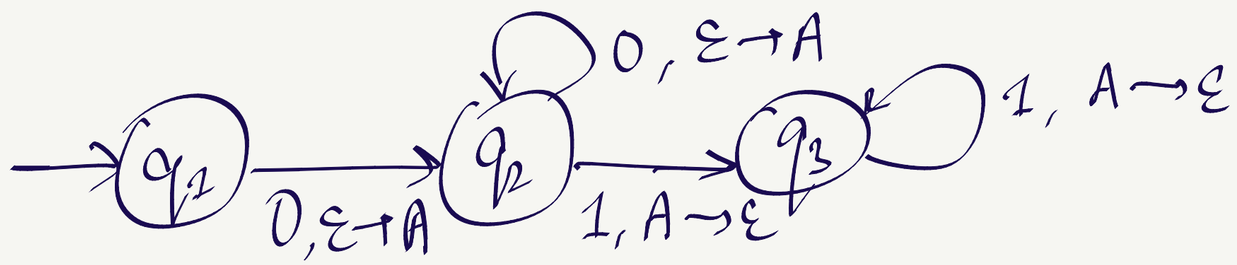
$$N_M = \{ \omega \mid \text{there are } u, v \in \Gamma^* \text{ s.t. } s\omega \xrightarrow[M]{*} urv \}$$

Suppose $\mathcal{L} = P_M$. Then, we say that $\mathcal{L}$ is recognized by M, and that $\mathcal{L}$ is Turing-recognizable, or recursively enumerable (r.e.)

If $\mathcal{L}$ is recognized by M, and in addition, $N_M = \{0,1\}^* \setminus P_M$, then we say that $\mathcal{L}$ is decided by M, and that $\mathcal{L}$ is decidable, or recursive

Decidability $\Rightarrow$ Turing-recognizability (but not the other way!)

$$L = \{ 0^n 1^n \mid n \geq 0 \}$$



$$M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, A\}, \delta, q_1, \phi)$$

A

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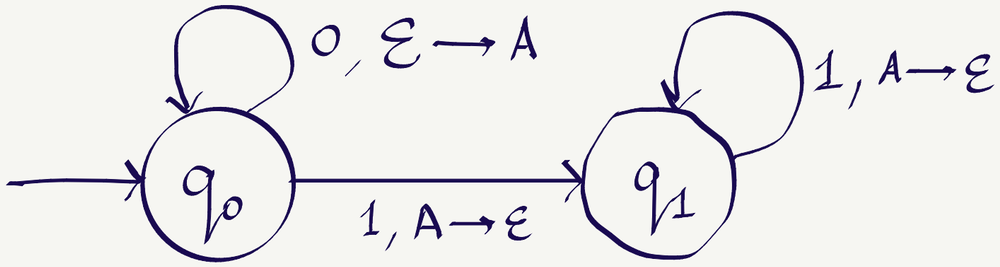
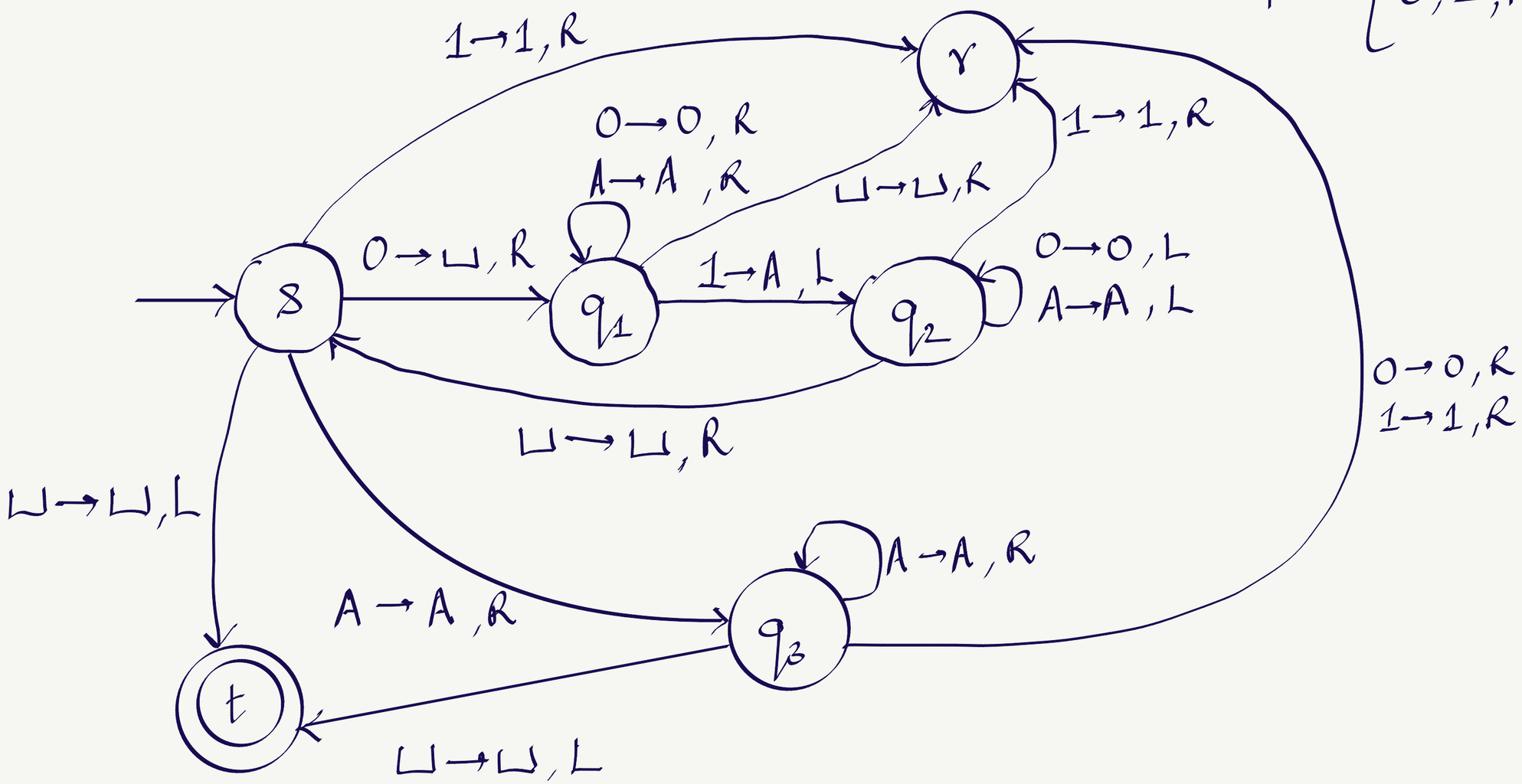
$$L = \{0^n 1^n \mid n \geq 0\}$$



$$Q = \{q_1, q_2, q_3, s, t, r\}$$

$$\Gamma = \{0, 1, A, \sqcup\}$$



Quiz