Turing Machines

Recall: Defined computability via URMs ("infinite RAM + assembly")

Computable = partial recursive functions = URM-computable

Non-computable = ?

Today: Other models of computation

Church-Turing thesis: The following models of computation are equivalent

→ The lambda calculus

→ Combinatory logic

→ Partial recursive functions

→ Post systems

→ Turing machines

A Turing machine is basically a finite-state automaton with a global tape

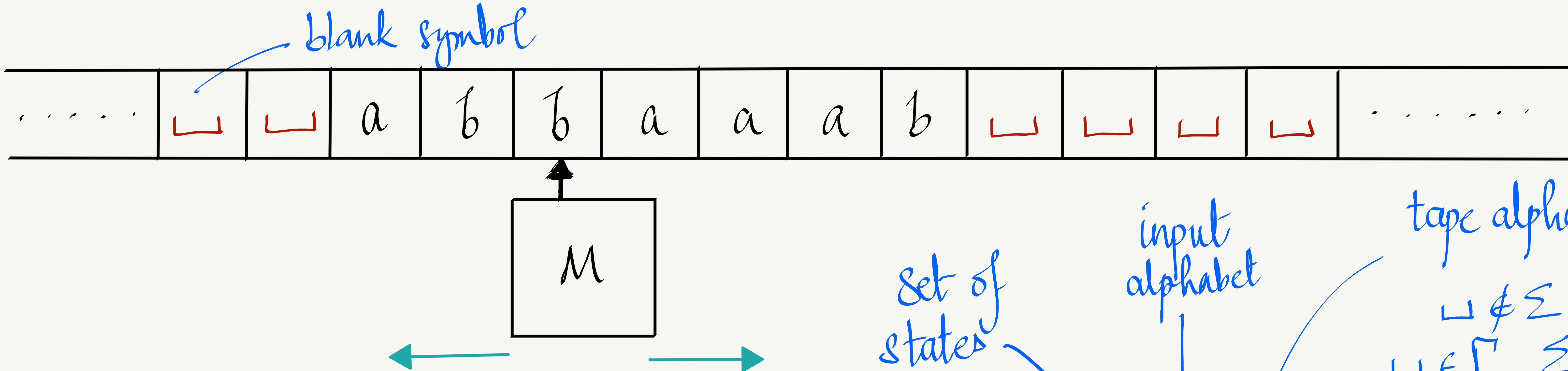
Each cell of the tape can contain a letter from a finite alphabet

Start out with the input string, and the rest of the tape is empty.

What can the machine do?

* Read off the tape * Write on the tape * Move the head left or right

At each step, the head scans some cell of the tape,
and the automaton is in some state, depending on which
the automaton can move to some (other) state,
the machine can write some new symbol on the tape cell, and
move the head one cell to the left or to the right



Set of states; input alphabet; tape alphabet

$\sqcup \notin \Sigma$

$\sqcup \in \Gamma$, $\Sigma \subseteq \Gamma$.

A deterministic* Turing Machine: $M = (Q, \Sigma, \Gamma, \delta, s, t, r)$

Start state; accept state; reject state

$s, t, r \in Q$.

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

current state; contents of current cell; new state; new contents of current cell; direction in which tape head should move

$Q \setminus \{t, r\}$

Input $\omega_1 \omega_2 \cdots \omega_n$, each $\omega_i \in \Sigma$

Written on n cells of the tape

The first blank signals the end of input.

As part of its operation, the following parameters change:

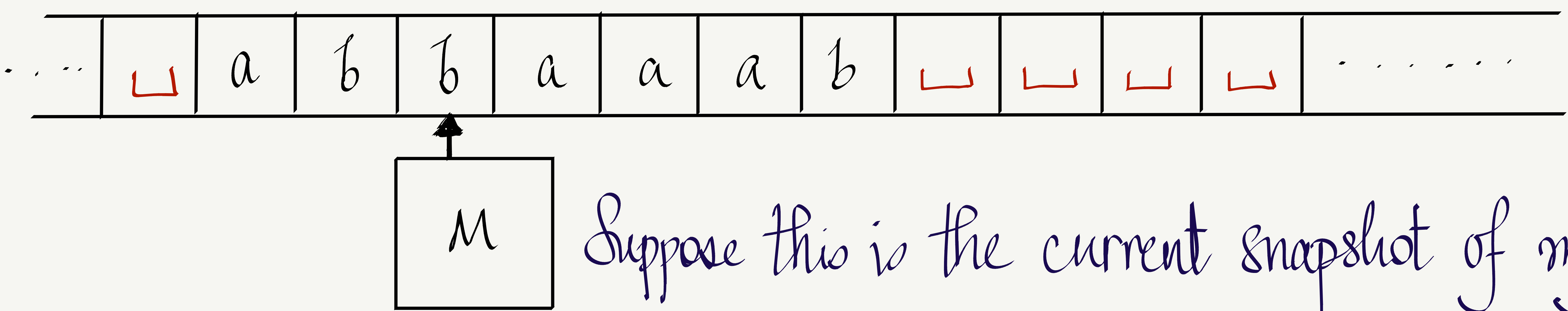
→ current state

→ current tape contents

→ location of tape head

These form a configuration

$(ab, b, aaab, q)$



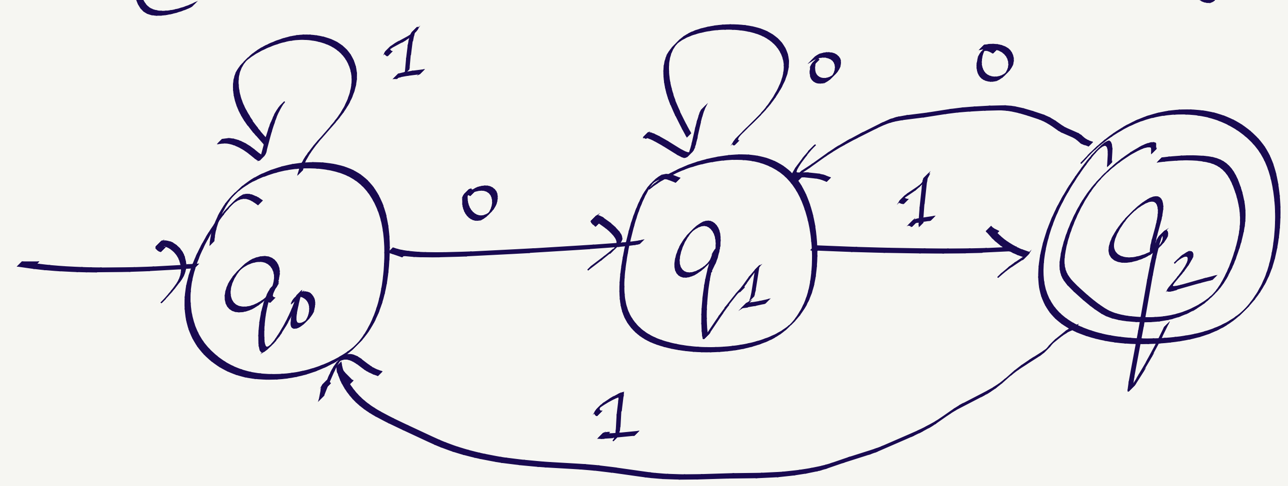
Suppose this is the current snapshot of my TM M, in state q.

We write the configuration as $ab\,q\,baaab$.

Formally, a configuration is uqv, where the machine is in state q, the current tape contents are uv, $u, v \in \Gamma^*$, and the head points to the cell containing the first letter of v.

Example :

$$\mathcal{L} = \{ \omega \mid \omega \text{ ends with } 01 \}$$



$$M = (Q, \Sigma, \Gamma, \delta, s, t, r)$$

$\Sigma = \{0, 1\}$ $\quad$ $\Gamma = \{0, 1, \sqcup\}$

$s = q_0$ $\quad$ $Q = \{q_0, q_1, q_2, t, r\}$

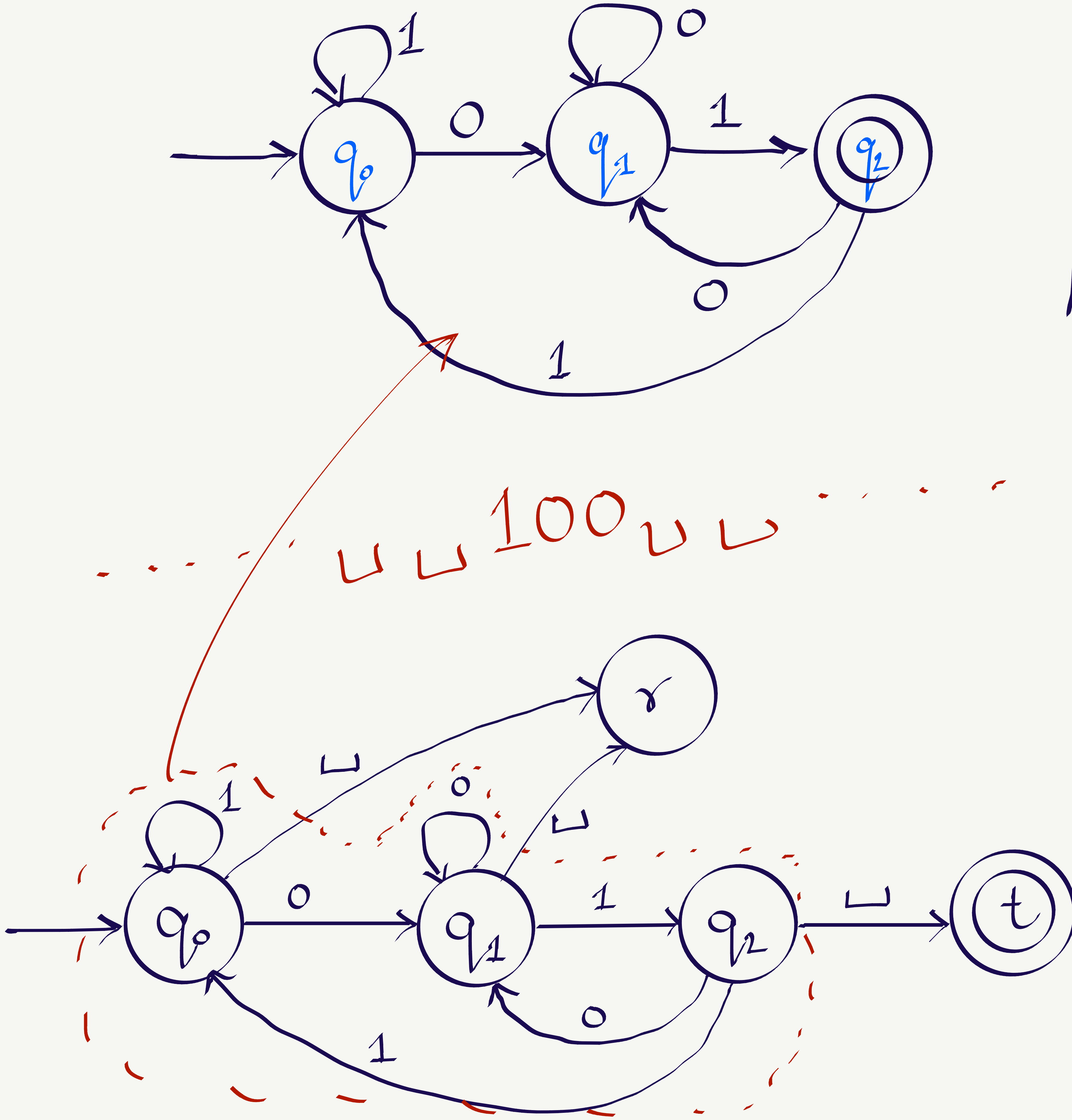
Example:

$$L = \{ w \mid w \text{ ends with } 01 \} \in \{0,1\}^* \qquad \Sigma = \{0,1\}$$

$$Q = \{q_0, q_1, q_2, t, r\}$$

$$\Gamma = \{0, 1, \sqcup\}$$

$$M = (Q, \Sigma, \Gamma, \delta, q_0, t, r)$$

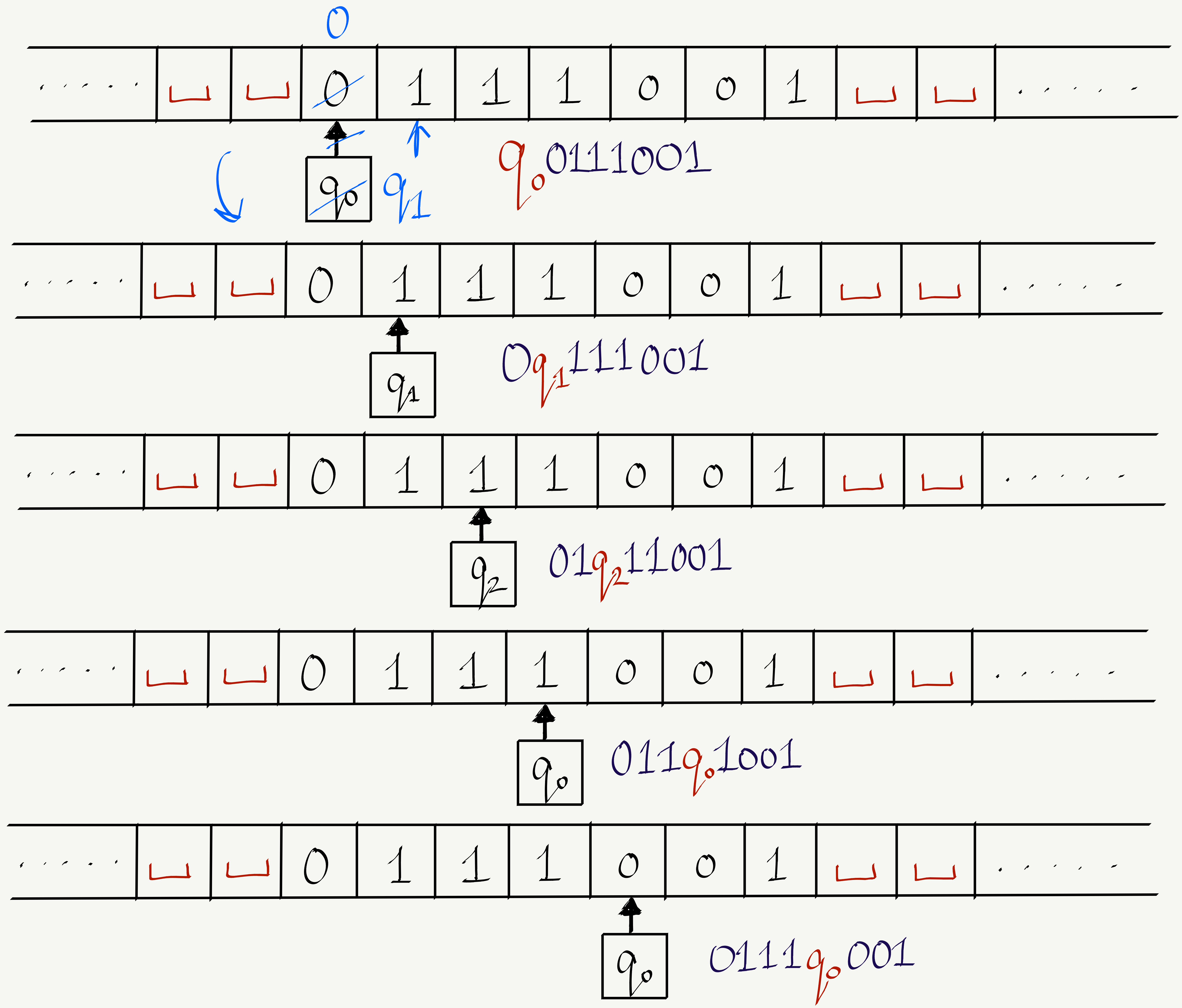


$$\delta(q_0, 0) = (q_1, 0, R)$$

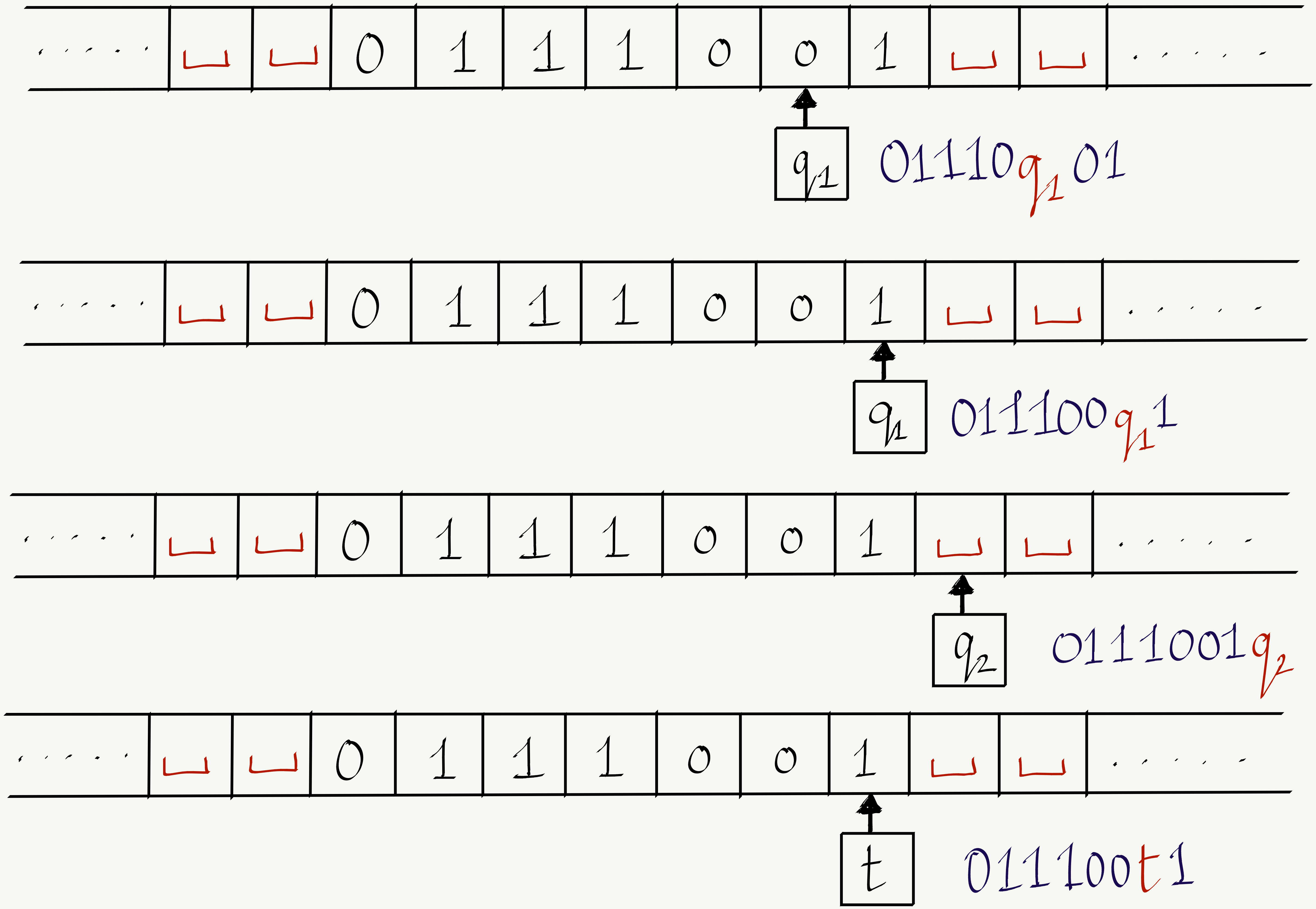
$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_2, \sqcup) = (t, \sqcup, L)$$



0 1 1 1 0 0 1
0
q_0 q_1
$q_0 0111001$
q_1
$0 q_1 111001$
q_2
$01 q_2 11001$
q_0
$011 q_0 1001$
q_0
$0111 q_0 001$



0 1 1 1 0 0 1
q1
01110q101
q1
011100q11
q2
0111001q2
t
011100t1

Let $u, v \in \Gamma^*$, and $a, b, c \in \Gamma$. Then,

$$uaqbv \xrightarrow[M]{1} uacq'v \quad \text{iff} \quad \delta(q,b) = (q', c, R), \quad \text{and}$$

$$uaqbv \xrightarrow[M]{1} uq'acv \quad \text{iff} \quad \delta(q,b) = (q', c, L)$$

The initial configuration is of the form sw, where w is the input.

w is accepted by M if $sw \xrightarrow[M]{*} utv$, for some $u, v \in \Gamma^*$.

accepting configuration

Similarly,

w is rejected by M if $sw \xrightarrow[M]{*} urv$, for some $u, v \in \Gamma^*$.

rejecting configuration