CYK Parsing

Recall: We saw how to convert any given CFG into

- Chomsky Normal form
  each rule $A \to c$ or $A \to BC$

- Greibach Normal form
  each rule $A \to cB_1 \dots B_k$, $k \geq 0$.

along with a rule $S \to \varepsilon$ if $\varepsilon \in \mathcal{L}$.

Today: Given a string ω and a CFG G, does G generate ω?

Need not necessarily have a PDA

Even with a PDA, not deterministic or efficient!

Can we somehow directly operate over the grammar?

Parsing algorithm: [Cocke, Younger, Kasami : 1961]

Main idea: Determine, for each substring x of ω,
the set of all non-terminals that generate x.

grammar in Chomsky Normal Form

Needs to do this systematically:

First check all substrings of length 1, ($A \to c$?)

then all substrings of length 2 or more

Consider every possible partitioning into two parts and match against $A \to BC$, where B generates the left part, and C generates the right part.

Doing this repeatedly?
Use dynamic programming!

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b \qquad C \to SB \qquad D \to SA$$

Consider a string $\omega = {}_{|}a_{|}b_{|}b_{|}b_{|}a_{|}a_{|}$ $\quad n = 6 = |\omega|$

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

ω_{ij} : substring of ω between markers i and j

Build a table with an entry for every (i, j), where $0 \leq i < j \leq n$.

j \ i	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

Fill $T(i, j)$ with the non-terminals that generate ω_{ij}

$j \backslash i$	0	1	2	3	4	5
1	A					
2		B				
3			B			
4				B		
5					A	
6						A

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

$$C \to SB \qquad D \to SA$$

$$\omega = {}_{0}\, a \,{}_{1}\, b \,{}_{2}\, b \,{}_{3}\, b \,{}_{4}\, a \,{}_{5}\, a \,{}_{6}$$

$\omega_{01} = a$ So $T(0,1) = \{A\}$. Similarly, $\omega_{12} = b$, so $T(1,2) = \{B\}$.

If there are multiple non-terminals which yield ω_{ij}, write them all in $T(i,j)$.

Now look at substrings of length 2.

j \ i	0	1	2	3	4	5
1	A					
2	S	B				
3		$\emptyset$	B			
4			$\emptyset$	B		
5				S	A	
6					$\emptyset$	A

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

$$C \to SB \qquad D \to SA$$

$$w = \underset{0}{|}\, a \,\underset{1}{|}\, b \,\underset{2}{|}\, b \,\underset{3}{|}\, b \,\underset{4}{|}\, a \,\underset{5}{|}\, a \,\underset{6}{|}$$

$w_{02} = ab$. Break this into two substrings of length 1, $\overset{w_{01}}{a}$, and $\overset{w_{12}}{b}$.

Look for all combinations of non-terminals which can yield these substrings, and look for a non-terminal which goes to this pair.

$T(0,1) = A$, $T(1,2) = B$, and $S \to AB$, so $T(0,2) = \{S\}$.

Fill in the diagonal below the top one this way.

j \ i	0	1	2	3	4	5
1	A					
2	S	B				
3	C	ϕ	B			
4		ϕ	ϕ	B		
5			ϕ	S	A	
6				D	ϕ	A

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

$$C \to SB \qquad D \to SA$$

$$\omega = {}_{0}|\, a \,|_{1}\, b \,|_{2}\, b \,|_{3}\, b \,|_{4}\, a \,|_{5}\, a \,|_{6}$$

$\omega_{03} = abb$. There are two ways to break this string:

ω_{01} — a and $\underbrace{bb}_{\omega_{13}}$, or $\underbrace{ab}_{\omega_{02}}$ and $\underbrace{b}_{\omega_{23}}$.

$T(0,1) = A$, $T(1,3) = \phi$

$T(0,2) = S$, $T(2,3) = B$

$C \to SB$, so $T(0,3) = C$

Do this for the diagonal.

$j \backslash i$	0	1	2	3	4	5
1	A					
2	S	B				
3	C	$\emptyset$	B			
4	$\emptyset$	$\emptyset$	$\emptyset$	B		
5		$\emptyset$	$\emptyset$	S	A	
6			S	D	$\emptyset$	A

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

$$C \to SB \qquad D \to SA$$

$$\omega = {}_0|\,a\,|_1\,b\,|_2\,b\,|_3\,b\,|_4\,a\,|_5\,a\,|_6$$

$\omega_{04} = abbb$ a, bbb or ab, bb or abb, b

None of these has any possible generating non-terminals.

$\omega_{26} = bbaa$ Only possibility: $\underbrace{b}_{B}$, $\underbrace{baa}_{D}$ $S \to BD$. So $T(2,6) = S$.

j \ i	0	1	2	3	4	5
1	A					
2	S	B				
3	C	$\emptyset$	B			
4	$\emptyset$	$\emptyset$	$\emptyset$	B		
5	$\emptyset$	$\emptyset$	$\emptyset$	S	A	
6	S	$\emptyset$	S	D	$\emptyset$	A

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

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$$w = {}_{0}|\, a \,|_{1}\, b \,|_{2}\, b \,|_{3}\, b \,|_{4}\, a \,|_{5}\, a \,|_{6}$$

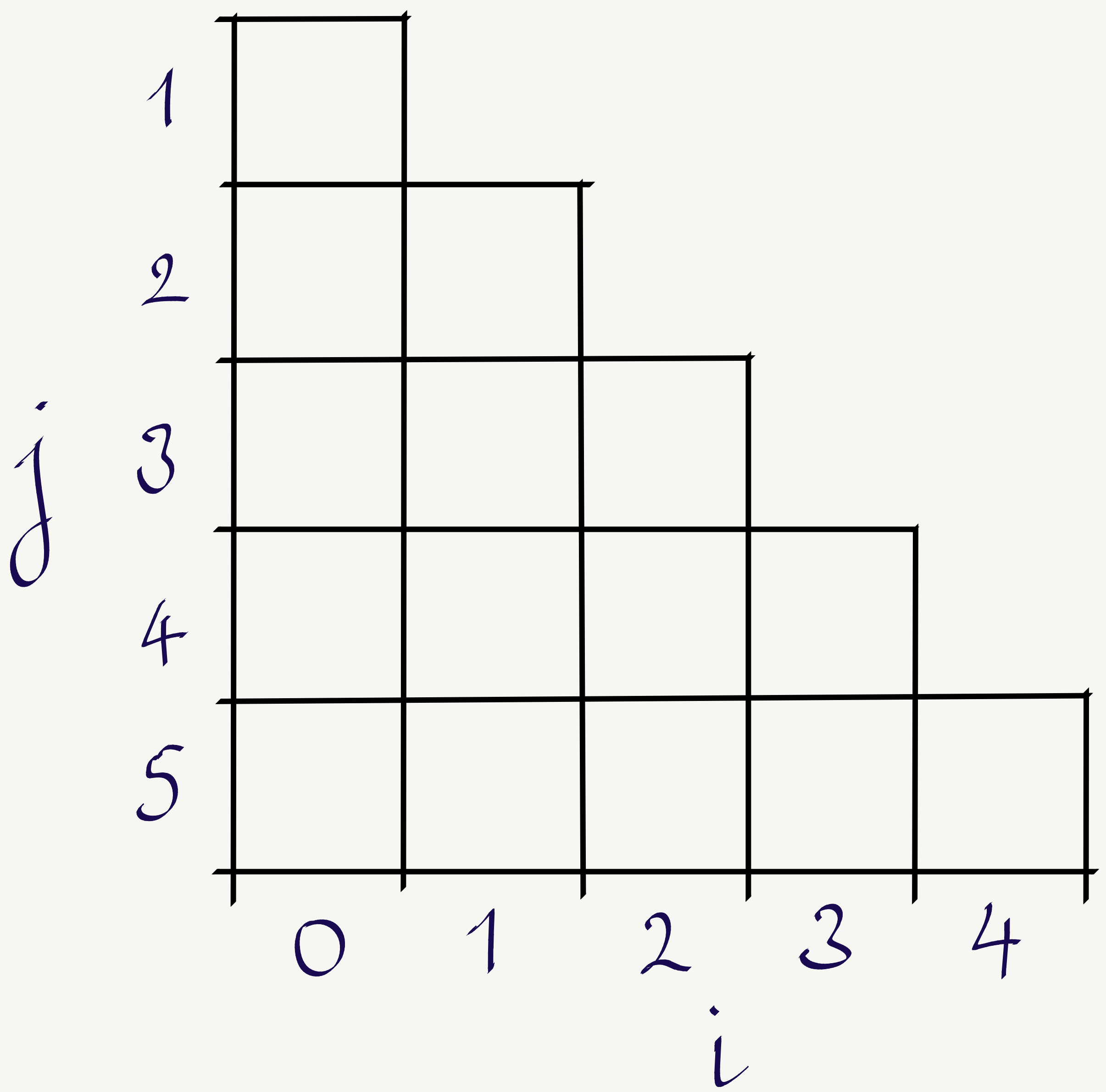
$w_{05} = abbba$

$w_{16} = bbbaa$

$w_{06} = w : \underbrace{ab}_{S}, \underbrace{bbaa}_{S} \qquad S \to SS$

Since $S \in T(0,6)$, this string is generated by the grammar.

```
for i := 0 to n − 1 do                      /* first do substrings of length 1 */
  begin
    T_{i,i+1} := ∅;                         /* initially assign the empty set */
    for A → a a production of G do
      if a = x_{i,i+1} then T_{i,i+1} := T_{i,i+1} ∪ {A}
  end;
for m := 2 to n do                          /* for each length m ≥ 2 */
  for i := 0 to n − m do                    /* for each substring of length m */
    begin
      T_{i,i+m} := ∅;                       /* initially assign the empty set */
      for j := i + 1 to i + m − 1 do        /* for all breaks of the string */
        for A → BC a production of G do
          if B ∈ T_{i,j} ∧ C ∈ T_{j,i+m}
            then T_{i,i+m} := T_{i,i+m} ∪ {A}
    end;
```



$$S \to AB \mid BA \mid SS \mid AC \mid BD$$

$$A \to a \qquad B \to b$$

$$C \to SB \qquad D \to SA$$

$$\omega = {}_{0}\,a\,{}_{1}\,b\,{}_{2}\,a\,{}_{3}\,b\,{}_{4}\,a\,{}_{5}$$

QUIZ